Pure Spinors in Classical and Quantum

经典与量子中的纯旋量

Supergravity

超引力

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Abstract

摘要

This is an overview of the method of pure spinor superfields, written for Handbook of Quantum Gravity, eds. C. Bambi, L. Modesto, and I. Shapiro. The main focus is on the use of the formalism in maximal supergravity on a flat background. The basics of pure spinor superfields, and their relation to standard superspace, are reviewed. The pure spinor superstring model of Berkovits is briefly discussed. Consequences for divergence properties of loop diagrams in maximal supergravity are restated. Some final remarks are made concerning desirable development of the theoretical framework.

本文是纯旋量超场方法的综述, 为《量子引力手册》撰写, 该书由 C. Bambi、L. Modesto 和 I. Shapiro 主编。本文主要关注该形式体系在平坦背景下极大超引力中的应用。本文回顾了纯旋量超场的基础, 及其与标准超空间的关系。对 Berkovits 的纯旋量超弦模型进行了简要讨论, 并重述了其对极大超引力圈图发散性质的结论。最后对该理论框架有待开展的发展进行了总结。

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Introduction

引言

Pure spinor superfield theory [1] provides a solution to the long-standing problem of covariant quantization of (Brink-Schwarz) superparticles [2,3] or (Green-Schwarz) superstrings [4] with manifest supersymmetry, or roughly equivalently, to the problem of finding off-shell superspace formulations of maximally supersymmetric field theories, including supergravity.

纯旋子超场理论 [1] 解决了一个长期悬而未决的问题: 对具有显式超对称性的 (布林克-施瓦茨) 超粒子 [2,3] 或 (格林-施瓦茨) 超弦 [4] 进行协变量子化, 大致等价地说, 它解决了寻找包括超引力在内的极大超对称场论的离-shell 超空间公式化问题。

Concretely, the difficulties with space-time supersymmetric particles and strings manifest themselves as a mixture of first- and second-class constraints in the same spinor. This is the famous κ -symmetry [5-7], which is necessary for the superparticle/superstring action to describe the dynamics of a $\frac{1}{2}$ -BPS object.

具体而言，时空超对称粒子和弦的困难体现为: 同一旋子中同时存在第一类和第二类约束。这就是著名的 κ 对称性 [5-7]，该对称性对超粒子/超弦作用量描述 $\frac{1}{2}$ -BPS 对象的动力学是必不可少的。

In the present overview, we will not start with these superparticle or superstring actions. Rather, the introduction of pure spinor variables will be motivated by the structure of the (on-shell) multiplets of maximal super-Yang-Mills theory (SYM) and supergravity (SG) in their traditional treatment on superspace. The relation of the pure spinor formulation to the Green-Schwarz superstring is explained in Ref. [8].

在本篇综述中, 我们不会从这些超粒子或超弦作用量出发, 而是从超空间上传统处理的极大超杨-米尔斯理论 (SYM) 和超引力 (SG) 的 (在壳) 多重态结构出发，引出纯旋子变量。纯旋子表述与格林-施瓦茨超弦的关系参见文献 [8]。

The basics of the formalism are laid out in section ”Pure Spinor Superfield Theory.” In section ” $D = 11$ Supergravity”, it is applied to supergravity, with maximal supergravity as main focus. A brief account of the pure spinor superstring theory of Berkovits is given in section ”Superstrings.” Quantum theory is sketched in section ”Quantum Theory,” and some convergence results for loop diagrams are restated. Finally, some remarks are made in section ”Remarks” concerning possible refinement and development of the formalism.

该形式体系的基础在“纯旋子超场理论”一节阐述。在“$D = 11$ 超引力”一节中，我们将它应用于超引力研究，重点关注极大超引力。“超弦”一节简要介绍了 Berkovits 的纯旋子超弦理论。“量子理论”一节概略描述了量子理论，重述了圈图的若干收敛结果。最后，“评注”一节对该形式体系可能的改进和发展做了补充讨论。

The technical level of the presentation is kept at a minimum. Instead, we aim at collecting results from the sources in the reference list and present them as concisely and coherently as possible while emphasizing concepts rather than techniques.

本文将技术细节的篇幅控制在最低，我们的目标是收集参考文献中已有的结论，尽可能简洁连贯地呈现它们，同时着重介绍概念而非技术。

Pure Spinor Superfield Theory

纯旋子超场理论

Before going into a more precise derivation of pure spinor superfield formulations of specific supersymmetric models, we would like to sketch what lies at the heart of the formalism. The supersymmetry algebra (which of course is a subalgebra of the super-Poincaré algebra) takes the generic form $\{Q_\alpha, Q_\beta\} = 2\gamma^a_{\alpha\beta}\partial_a$. Here, α is some (possibly multiple) spinor index, and $Q_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\gamma^a\theta)_\alpha\partial_a$. Covariant fermionic derivatives $D_\alpha = \frac{\partial}{\partial\theta^\alpha} - (\gamma^a\theta)_\alpha\partial_a$ satisfy $\{Q_\alpha, D_\beta\} = 0$. They anticommute among themselves as $\{D_\alpha, D_\beta\} = -2\gamma^a_{\alpha\beta}\partial_a-$ flat superspace in endowed with torsion $T_{\alpha\beta}{}^a = 2\gamma^a_{\alpha\beta}$.

在对特定超对称模型的纯旋子超场表述进行更精确的推导之前，我们先概要介绍这个形式体系的核心内容。超对称代数(它当然是超庞加莱代数的一个子代数)的一般形式为 $\{Q_\alpha, Q_\beta\} = 2\gamma^a_{\alpha\beta}\partial_a$ 。此处，α 是某个(可以是多个)旋子指标，且 $Q_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\gamma^a\theta)_\alpha\partial_a$ 。协变费米导数 $D_\alpha = \frac{\partial}{\partial\theta^\alpha} - (\gamma^a\theta)_\alpha\partial_a$ 满足 $\{Q_\alpha, D_\beta\} = 0$ 。它们之间的反对易关系为 $\{D_\alpha, D_\beta\} = -2\gamma^a_{\alpha\beta}\partial_a -$ ，平坦超空间带有挠率 $T_{\alpha\beta}{}^a = 2\gamma^a_{\alpha\beta}$ 。

Suppose we introduce a bosonic spinor λ^α subject to the constraint

假设我们引入一个满足如下约束的玻色旋子 λ^α

$$(\lambda\gamma^a\lambda) = 0. \tag{1}$$

Then we may form a fermionic operator

随后我们可以构造一个费米算符

$$Q = \lambda^\alpha D_\alpha \tag{2}$$

which, thanks to the constraint on λ , is nilpotent: $Q^2 = 0$.

借助 λ 满足的约束，该算符是幂零的: $Q^2 = 0$ 。

It seems meaningful to consider the cohomology of Q , acting on functions of x , θ , and λ . This cohomology is guaranteed to be supersymmetric, since Q anticom-mutes with the supersymmetry generators. It thus describes some supermultiplet. As it turns out, any linear supermultiplet in any dimension may be obtained this way. In the case of on-shell multiplets, the virtue of the formalism is even greater, since it seems to offer a natural way to an off-shell formulation by relaxation of the linear "equation of motion" $Q\Psi = 0$. The correspondence will be made more precise below, first for $D = 10$ super-Yang-Mills theory and later for $D = 11$ supergravity.

我们可以考虑作用在 x 、θ 和 λ 函数上的 Q 上同调，这是有明确意义的。由于 Q 与超对称生成元反对易，因此该上同调必然是超对称的，它描述了某个超多重态。事实证明，任意维度下的任意线性超多重态都可以通过这种方式得到。对于在壳多重态，该形式体系的优势更为显著，因为通过放宽线性“运动方程” $Q\Psi = 0$ ，它似乎为离壳表述提供了一条自然途径。我们会在下文把对应关系讲得更清楚，先讨论 $D = 10$ 超杨-米尔斯理论，再讨论 $D = 11$ 超引力。

A word of caution: We will refer to a spinor λ subject to the constraint (1) as a "pure spinor." This is a slight misuse of the mathematical terminology. A pure spinor, in the sense of Cartan [9], is a chiral spinor in even dimension $D = 2n$, constrained to lie in the minimal Spin(2n) orbit of the spinor module. This implies that if the Dynkin label of the spinor module in question is $(0\ldots01)$, monomials of degree of homogeneity p in λ belong to the single module $(0\ldots0p)$. The concept of a pure spinor is not defined in other cases, neither for odd dimensions nor for extended supersymmetry. In certain cases, our constrained spinors coincide with Cartan pure spinors. This happens notably in $D = 10$. There, the spinor bilinears are a vector (10000) and a self-dual 5-form (00002), and the constraint in the vector immediately puts λ in the minimal orbit. In other situations, for example, $D = 11$, which we will encounter later, where the symmetric spinor bilinears are a vector, a 2-form and a 5-form, the vector constraints puts λ in (a completion of) an intermediate orbit, which is not the minimal one.

需要提醒一点: 我们将满足约束 (1) 的旋子 λ 称为“纯旋子”。这是对数学术语的一种轻度误用。在嘉当 [9] 的定义中，纯旋子是偶数维 $D = 2n$ 中的手征旋子，满足位于旋子模的最小 Spin(2n) 轨道上的约束。这意味着，如果讨论的旋子模的 Dynkin 标记为 $(0\ldots01)$，则 λ 上齐次次数为 p 的单项式都属于单个模 $(0\ldots0p)$。纯旋子的概念在其他情形下都没有定义，无论是奇数维还是扩展超对称的情形。在某些情况下，我们这里带约束的旋子与嘉当纯旋子是一致的，最典型的例子就是 $D = 10$ 中的情形。在该情形下，旋子双线性是一个矢量 (10000) 和一个自对偶 5-形式 (00002)，矢量上的约束直接将 λ 限制在最小轨道中。在其他情形下，例如我们之后会遇到的 $D = 11$，对称旋子双线性包含一个矢量、一个 2-形式和一个 5-形式，矢量约束将 λ 限制在中间轨道 (的完备化) 中，这个轨道不是最小轨道。

From Superspace to Pure Spinor Superspace

从超空间到纯旋量超空间

Although we ultimately aim at addressing supergravity, the introduction of pure spinor superspace is much simpler in the setting of super-Yang-Mills theory [10], first treated in superspace in Ref. [11].

尽管我们最终目标是研究超引力，纯旋量超空间的引入在超杨-米尔斯理论框架下要简单得多 [10]，该理论最早在参考文献 [11] 中于超空间框架下被研究。

Flat (10|16)-dimensional superspace, appropriate for $D = 10$ super-Yang-Mills, has coordinates $Z^M = (x^m, \theta^\mu)$. There is no metric on superspace, but a super-vielbein ${E_M}^A$. The Lorentz frame indices $A = (a, \alpha)$ consist of a Lorentz vector and a chiral spinor. The nonvanishing superspace torsion is

适用于 $D = 10$ 超杨-米尔斯的平坦 (10|16) 维超空间具有坐标 $Z^M = (x^m, \theta^\mu)$。超空间本身没有度规，但存在超 vielbein ${E_M}^A$。洛伦兹标架指标 $A = (a, \alpha)$ 包含一个洛伦兹矢量和一个手征旋量。非零的超空间挠率为:

$$
{T_{\alpha\beta}}^a = 2\gamma^a_{\alpha\beta}, \tag{3}
$$

where the components are converted to Lorentz frame using the inverse vielbein: ${T_{BC}}^A = {\left(E^{-1}\right)_C}^N {\left(E^{-1}\right)_B}^M {T_{MN}}^A$.

其中各分量通过逆 vielbein 转换到洛伦兹标架: ${T_{BC}}^A = {\left(E^{-1}\right)_C}^N {\left(E^{-1}\right)_B}^M {T_{MN}}^A$ 。

Let us now recollect some known facts about $D = 10$ super-Yang-Mills. A connection on superspace, taking values in the adjoint of some gauge group, is written $A = dZ^M A_M = dZ^M {E_M}^A A_A$. There is a priori two superfields, $A_a(x, \theta)$ (bosonic, dimension 1) and $A_\alpha(x, \theta)$ (fermionic, dimension $\frac{1}{2}$) (As is standard, dimension is in terms of powers of inverse length.). The field strength is $F = dA + A \wedge A$, and due to the presence of torsion, we have

现在我们回顾一下关于 $D=10$ 超杨-米尔斯的已知结论。取值在某个规范群伴随表示的超空间联络可写为 $A=dZ^M A_M = dZ^M E_M{}^A A_A$。原本存在两个超场: $A_a(x,\theta)$ (玻色子，量纲 1) 和 $A_\alpha(x,\theta)$ (费米子，量纲 $\frac{1}{2}$)(按照惯例，量纲按长度倒数的幂次计算)。场强为 $F=dA+A\wedge A$，由于挠率的存在，我们有:

$$F_{\alpha\beta} = 2D_{(\alpha}A_{\beta)} + 2A_{(\alpha}A_{\beta)} + 2\gamma^a_{\alpha\beta}A_a. \tag{4}$$

The symmetric product of two spinors can be decomposed into a vector $F_a = \frac{1}{16}\gamma_a^{\alpha\beta}F_{\alpha\beta}$ and a self-dual 5-form $F_{abcde} = \frac{1}{2\cdot 5!\cdot 16}\gamma_{abcde}^{\alpha\beta}F_{\alpha\beta}$. Obviously, from Eq. (4), setting $F_a=0$ expresses A_a in terms of A_α , leaving only the latter as an independent superfield. This goes under the name of ”conventional constraint.” Note that it is natural, since $F_{\alpha\beta}$ has dimension 1, and there are no physical and gauge-covariant fields of this dimension in the supermultiplet we want to derive (the spinor χ^α has dimension $\frac{3}{2}$ and the field strength F_{ab} dimension 2). For this reason, it is tempting to also set the five-form part F_{abcde} to zero. However, doing this turns out to put the theory on shell. It indeed describes the on-shell $D=10$ super-Yang-Mills multiplet. The constraint in question is physical, rather than conventional. Details can be found, for example, in Refs. [12,13]. It is of course also well known that the supersymmetry transformations of the component fields work (”close,” modulo gauge transformations) only when the equations of motion are satisfied. There is no known set of auxiliary fields consistent with the Euler-Lagrange equations of an action that remedies this. This statement can indeed be sharpened. Auxiliary fields are fields that complete the physical ones into an off-shell linear module of the supersymmetry algebra. This cannot be done without completely relaxing the constraints on the superfield A_α off-shell.

两个旋量的对称积可以分解为一个矢量 $F_a = \frac{1}{16}\gamma_a^{\alpha\beta}F_{\alpha\beta}$ 和一个自对偶 5-形式 $F_{abcde} = \frac{1}{2\cdot 5!\cdot 16}\gamma_{abcde}^{\alpha\beta}F_{\alpha\beta}$。显然，由式 (4)，令 $F_a=0$ 后即可将 A_a 用 A_α 表示，仅留下后者作为独立超场。这就是所谓的“常规约束”。请注意这是自然的，因为 $F_{\alpha\beta}$ 的量纲为 1，而我们要推导的超多重态中不存在该量纲的物理规范协变场 (旋量 χ^α 量纲为 $\frac{3}{2}$，场强 F_{ab} 量纲为 2)。因此，人们很自然会进一步将五形式部分 F_{abcde} 也置零。但这样做最终会将理论推到在壳条件。这确实描述了在壳 $D=10$ 超杨-米尔斯多重态。这里的约束是物理约束，而非常规约束。相关细节可以在参考文献 [12,13] 中找到。当然，众所周知分量场的超对称变换仅当运动方程满足时才成立 (模规范变换“封闭”)。目前还没有已知的、与作用量的欧拉-拉格朗日方程相容的辅助场集合能解决这个问题。这个结论实际上可以进一步明确: 辅助场是用来将物理场补全为超对称代数的脱壳线性模的场。如果不完全放松超场 A_α 的约束使其脱壳，就无法完成这一补全。

This observation prompted Nilsson [12] to first draw the (correct) conclusion that in order to go off-shell one needs to relax the 5-form part of $F_{\alpha\beta}=0$. And this is what pure spinor superfield theory naturally does, as we will see. Indeed, the equations of motion in the pure spinor superfield description of $D=10$ super-Yang-Mills theory will be $\gamma_{(5)}^{\alpha\beta}F_{\alpha\beta}=0$.

这一发现促使 Nilsson[12] 首先得出了 (正确的) 结论: 为了得到脱壳理论，需要放宽 $F_{\alpha\beta}=0$ 的 5-形式部分约束。正如我们即将看到的，纯旋子超场理论自然满足了这一点。事实上，$D=10$ 超杨-米尔斯理论在纯旋子超场描述下的运动方程就是 $\gamma_{(5)}^{\alpha\beta}F_{\alpha\beta}=0$。

This structure was found when searching for deformations of the equations of motion for maximally supersymmetric super-Yang-Mills [13-15]. Introduce a bosonic spinor λ^α , subject to the constraint $(\lambda\gamma^a\lambda)=0$

. A function Ψ of x, θ , and λ , expanded in powers in λ , is (There is no other way of dealing with the λ dependence, since no scalar is encountered at any power of λ .)

> 该结构是在研究最大超对称超杨-米尔斯运动方程的形变时发现的 [13-15]。引入满足约束 $(\lambda\gamma^{a}\lambda) = 0$ 的玻色旋子 λ^{α} ，以及关于 x, θ 和 λ 、按 λ 幂次展开的函数 Ψ ，(由于 λ 的任意次幂都不出现标量，没有其他方法处理对 λ 的依赖关系。)

$$\Psi(x, \theta, \lambda) = C(x, \theta) + \lambda^{\alpha} A_{\alpha}(x, \theta) + \frac{1}{2}\lambda^{\alpha}\lambda^{\beta} B_{\alpha\beta}(x, \theta) + \ldots \tag{5}$$

Now, acting with the ”BRST operator” Q gives

> 现在，作用”BRST 算符” Q 后可得

$$Q\Psi = \lambda^{\alpha} D_{\alpha} C + \lambda^{\alpha}\lambda^{\beta} D_{\alpha} A_{\beta} + \ldots \tag{6}$$

The linearized equations of motion are encoded as Ψ being Q -closed, and gauge transformations correspond to Q -exact functions. It also immediately follows that for the specific choice $\Psi = \lambda^{\alpha} A_{\alpha}$,

> 线性化运动方程可以表述为 Ψ 是 Q 闭的，而规范变换对应 Q 恰当函数。由此可以直接得到，对特定选择 $\Psi = \lambda^{\alpha} A_{\alpha}$ ，

$$Q\Psi + \Psi^2 = \lambda^{\alpha}\lambda^{\beta} F_{\alpha\beta}. \tag{7}$$

The full nonlinear equations of motion are encoded as

> 完整的非线性运动方程可以表述为

$$Q\Psi + \Psi^2 = 0. \tag{8}$$

One should think of λ as a ghost variable, which explains it being bosonic, although it is a spinor. Then Ψ should also be assigned ghost number 1, so that A_{α} has ghost number 0 . The above shows that the cohomology of Q in the ghost number 0 sector is precisely the linear super-Yang-Mills multiplet. One should also make sure that there is no essential cohomology in other ghost numbers (powers of λ) . This can be done as follows:

> 我们应当将 λ 视为鬼场变量，这也解释了它为什么是玻色型的——尽管它是一个旋子。那么 Ψ 也需要被赋予鬼数 1，这样 A_{α} 的鬼数就是 0。上述内容表明，Q 在鬼数 0 区的上同调恰好就是线性超杨-米尔斯多重态。我们还需要确认其他鬼数区 (λ) 的幂次区) 没有非平凡上同调，验证过程如下:

In order to investigate the cohomology, we will do it in two steps: first, we find the zero-mode (i.e., x -independent) cohomology. It will correspond to fields in a component formulation. Then, in the second step, these fields will, in the full cohomology, be related by differential operators constructed from $\frac{\partial}{\partial x}$. The procedure is not presented as a mathematical proof here; a fuller account can be found in Refs. [1, 16, 17] .

为了研究上同调，我们分两步进行: 首先，我们求解零模 (即不依赖 x 的) 上同调，它对应分量表述中的场。然后第二步，这些场在完整上同调中由 $\frac{\partial}{\partial x}$ 构造的微分算子联系起来。本文不将该过程作为数学证明展开，完整阐述可参见文献 [1, 16, 17]。

The zero-mode cohomology of Q is the cohomology of $\lambda^\alpha \frac{\partial}{\partial\theta^\alpha}$. Had λ been unconstrained, the only cohomology would have been the constant one. Now, when λ is constrained, the problem is algebraic, and the result is reflected in the partition function of λ . Encode the power of λ in a variable t . Then the partition function, taking values in the representation ring, is

Q 的零模上同调就是 $\lambda^\alpha \frac{\partial}{\partial\theta^\alpha}$ 的上同调。若 λ 不受约束，唯一的上同调就是常数上同调。当 λ 带约束时，问题化为代数问题，结果由 λ 的配分函数反映。将 λ 的幂次编码进变量 t ，则取值于表示环的配分函数为

$$Z(t) = (00000) \oplus (00001)\, t \oplus (00002)\, t^2 \oplus (00003)\, t^3 + \ldots \tag{9}$$

It is straightforward to factor out the dependence of an unconstrained spinor, which we write as

我们很容易因式分解出无约束旋子的依赖关系，写作

$$(1-t)^{-(00001)} = (00000) \oplus (00001)\, t \oplus \vee^2 (00001)\, t^2 \oplus \vee^3 (00001)\, t^3 + \ldots$$

(10)

($\vee^p$ is the p -fold totally symmetric product). We then obtain

($\vee^p$ 是 p 重全对称积。由此我们得到

$$Z(t) = (1-t)^{-(00001)} \tag{11}$$

$$\otimes \left[(00000) \ominus (10000)\, t^2 \oplus (00010)\, t^3 \ominus (00001)\, t^5 \oplus (10000)\, t^6 \ominus (00000)\, t^8\right].$$

Each term inside the square brackets represents a component field. They are, in order of appearance, the ghost c , the fields in the physical multiplet A_a and χ^α , the antifields $\chi^\star_\alpha$ and $A^{\star a}$, and the ghost antifield $c^\star$. Each of them appears in the zero-mode cohomology as some function of θ and λ , for example, the physical gauge field appears as $(\lambda\gamma^a\theta) A_a$.

方括号内的每一项都代表一个分量场，按出现顺序依次为鬼场 c 、物理多重态中的场 A_a 和 χ^α 、反场 $\chi^\star_\alpha$ 和 $A^{\star a}$ ，以及鬼反场 $c^\star$ 。每个场作为 θ 和 λ 的某个函数出现在零模上同调中，例如物理规范场可表示为 $(\lambda\gamma^a\theta) A_a$ 。

Going back to the full cohomology of Q , it will relate the component fields, now x -dependent, with differential operators. The proper mathematical tool for this procedure is that of homotopy transfer (see Ref. [16]). It is straightforward to show that the action of Q is indeed that of the BRST operator of the component fields and antifields of the super-Yang-Mills theory. The cohomology consists precisely of the linearized physical (on-shell) fields and a ghost zero mode.

回到 Q 的整个上同调，它会把现在依赖于 x 的分量场与微分算子联系起来。该过程适用的数学工具是同伦转移 (参见参考文献 [16])。我们可以直接证明，Q 的作用恰好对应超杨-米尔斯理论中分量场和反场的 BRST 算符作用。该上同调恰好由线性化物理 (在壳) 场和一个鬼零模构成。

It looks tempting to try to derive Eq. (8) from a Chern-Simons-like action. This can indeed be done, leading to the appropriate off-shell formulation, but requires the machinery of the following subsection. It is also clear from the nature of the cohomology that such an action should be regarded as a Batalin-Vilkovisky (BV) action [18], containing ghosts, fields, and their antifields.

我们很容易尝试从类陈省身-西蒙斯作用量推导式 (8)。这确实是可行的，能够得到恰当的离壳表述，但需要用到下一小节的方法。另外从该上同调的性质也可以明确，这类作用量应当被视为巴塔林-维尔可维斯基 (BV) 作用量 [18]，包含鬼、场及其反场。

Non-minimal Variables, Integration, and BV Actions

非最小变量、积分与 BV 作用量

In order to write an action that reproduces the equations of motion of the previous subsection, one needs an integration over the pure spinor λ . In addition, it should (when one also includes integration over θ) pick up the top zero-mode cohomology, i.e., the top component of the component super-Yang-Mills BRST complex, corresponding to the ghost antifield. This cohomology sits at $\lambda^3\theta^5$. One is in the seemingly problematic situation of needing a residue-like measure, in the sense of picking a certain component, while on the other hand having a series expansion that contains only positive powers of λ . Such a measure is clearly degenerate, and not useful.

为了写出能重现上一小节运动方程的作用量，我们需要对纯旋子 λ 积分。此外，它 (当我们同时包含对 θ 的积分时) 应当提取顶部零模上同调，也就是对应反鬼的分量形式超杨-米尔斯 BRST 复形的顶部分量。该上同调位于 $\lambda^3\theta^5$ 。我们遇到了一个看似有问题的情况: 一方面需要一个类似留数的测度来选取特定分量，另一方面级数展开只包含 λ 的正幂次。这样的测度显然是退化的，没有用处。

This problem was solved in Ref. [19], using what is known as a non-minimal set of variables. In addition to the pure spinor λ^α , one introduces a conjugate pure spinor $\bar\lambda_\alpha$. In order not to disturb the cohomology, an equal number of additional fermions r_α , which are pure with respect to $\bar\lambda$: $\left(\bar\lambda\gamma^a r\right)=0$. We identify r_α as $d\bar\lambda_\alpha$ and products of r s with wedge product of $d\bar\lambda$ ’ s. Then, the modified non-minimal BRST operator

文献 [19] 使用所谓的非最小变量组解决了这个问题。除纯旋子 λ^α 外，人们引入了共轭纯旋子 $\bar\lambda_\alpha$ 。为了不破坏上同调，还引入了相同数量的附加费米子 r_α ，它们相对于 $\bar\lambda$: $\left(\bar\lambda\gamma^a r\right)=0$ 是纯的。我们将 r_α 认作 $d\bar\lambda_\alpha$ ，并将 r s 的乘积认作 $d\bar\lambda$ 的外积。于是，修改后的非最小 BRST 算子

$$Q=(\lambda D)+\bar\partial, \tag{12}$$

where $\bar\partial=d\bar\lambda_\alpha\frac{\partial}{\partial\bar\lambda_\alpha}$ is the Dolbeault operator, has the same cohomology as the minimal one previously considered.

其中 $\bar{\partial} = d\bar{\lambda}_\alpha \frac{\partial}{\partial \bar{\lambda}_\alpha}$ 是多尔博尔特算子，其上同调与我们之前讨论的最小情形的上同调完全相同。

The pure spinor space is a (non-compact) Calabi-Yau space [20]. It possesses a holomorphic top form, in this case an 11-form Ω . The schematic form of this Calabi-Yau form is

纯旋子空间是 (非紧) 卡拉比-丘空间 [20]。它具有一个全纯顶部形式，在此情形下是 11-形式 Ω 。该卡拉比-丘形式的概要形式为

$$\Omega \sim \lambda^{-3}(d\lambda)^{11}. \tag{13}$$

For detailed expressions, see Refs. [1, 19, 20].

具体表达式参见文献 [1, 19, 20]。

Remember that the pure spinor field Ψ now depends on $x, \theta, \lambda, \bar{\lambda}$, and $d\bar{\lambda}$. The last dependence is seen as Ψ being an antiholomorphic cochain. One may try an integration measure

记住现在纯旋子场 Ψ 依赖于 $x, \theta, \lambda, \bar{\lambda}$ 和 $d\bar{\lambda}$ 。最后这个依赖关系可以理解为 Ψ 是一个反全连上链。我们可以尝试如下积分测度

$$\int [dZ]\, f = \int d^{10}x \int d^{16}\theta \int \Omega \wedge f, \tag{14}$$

where the last integral is over the pure spinor Calabi-Yau space. This measure is nondegenerate and carries ghost number -3 as desired, due to the λ^{-3} in Ω . However, the cohomologies we encountered have representatives which are 0- forms, so any pair of such functions seems to have a vanishing scalar product. On the other hand, the pure spinor space is a non-compact cône, so integrals naïvely diverge at large radius. This " $0 \times \infty$ " structure can be regularized [19] to yield finite results. The trick is to observe that the behavior on pure spinor space is topological and to insert a Q -invariant regularization [19, 21] $e^{-t\{Q,\chi\}}$ for some fermion χ . Such a regulator will give t -independent results. If one chooses $\chi = \theta^\alpha \bar{\lambda}_\alpha$, one gets a factor

其中最后一个积分是对纯旋子卡拉比-丘空间的积分。由于 Ω 中的 λ^{-3} ，该测度是非退化的，且恰好具有我们需要的鬼数-3。但我们遇到的上同调的代表都是 0-形式，因此任何一对这样的函数的标量积似乎都是零。另一方面，纯旋子空间是非紧锥，所以朴素积分在大半径处发散。这种" $0 \times \infty$ " 结构可以通过正则化 [19] 得到有限结果。技巧在于注意到纯旋子空间上的行为是拓扑的，只需对某个费米子 χ 插入一个 Q 不变的正则化因子 [19, 21] $e^{-t\{Q,\chi\}}$ 。这种正则化会给出与 t 无关的结果。如果选取 $\chi = \theta^\alpha \bar{\lambda}_\alpha$ ，就会得到一个因子

$$e^{-t\{Q,\chi\}} = e^{-t\left(\left(\lambda\bar{\lambda}\right)+\left(\theta d\bar{\lambda}\right)\right)}. \tag{15}$$

The first factor makes integrals convergent at large radius. The second one contains terms up to $\theta^{11}\left(d\bar{\lambda}\right)^{11}$. When integrated with a 0-form, it will pick up a component at θ^5 . This regulated measure is exactly what is needed. We can think of it as an operator that localizes the integral to the vicinity of the tip $\lambda = 0$ of the pure spinor cône. Alternatively, the basis for cohomology can be chosen to include such factors, and then no regularization of the measure is necessary.

第一个因子使大半径处的积分收敛。第二个因子包含最高到 $\theta^{11}\left(d\bar{\lambda}\right)^{11}$ 的项。当它和一个 0-形式积分时，会提取出 θ^5 处的分量。这种正则化后的测度恰好满足需求。我们可以将它看作一个算子，把积分局域化到纯旋子锥顶点 $\lambda = 0$ 的邻域。或者，我们也可以选择上同调基时就包含这类因子，这样就不需要对测度做正则化了。

Now, a Chern-Simons-like BV action for $D = 10$ can be written as [22, 23]

现在，我们可以将对应 $D = 10$ 的类似陈-西蒙斯的 BV 作用量写为 [22, 23]

$$S = \int [dZ]\,\mathrm{tr}\left(\frac{1}{2}\Psi Q\Psi + \frac{1}{3}\Psi^3\right). \tag{16}$$

Note that the action only contains a cubic interaction term, while the component F^2 contains quartic interactions. A component action can be derived by homotopy transfer [16] or, put more mundanely, the higher-order interactions arise from repeated use of the equations of motion. See also Ref. [24]. This property, that the supersymmetric action is of lower order than the component action, becomes even more pronounced when we turn to supergravity in the following section.

请注意，该作用量仅包含三次相互作用项，而分量场 F^2 包含四次相互作用。分量作用量可通过同伦转移导出 [16]；说得更通俗些，高阶相互作用是反复运用运动方程得到的。另见文献 [24]。超对称作用量的阶数低于分量作用量这一性质，在下一节我们讨论超引力时会更加明显。

The superfield Ψ is self-conjugate with respect to the BV anti-bracket:

超场 Ψ 相对于 BV 反括号是自共轭的:

$$(A, B) = \int A \frac{\overleftarrow{\delta}}{\delta\Psi(Z)}\,[dZ]\,\frac{\overrightarrow{\delta}}{\delta\Psi(Z)} B. \tag{17}$$

Then it is straightforward to show that the classical master equation $(S, S) = 0$ is satisfied.

那么很容易就能证明经典主方程 $(S, S) = 0$ 成立。

Other Models

其他模型

Any supermultiplet can be derived as the cohomology in a pure spinor superfield. In many cases, the zero-mode cohomology is such that the corresponding fields define a component BV complex and an integration can be defined. Situations where this does not happen is, for example, when self-dual tensors are contained in the supermultiplet, such as the $N = (2, 0)$ multiplet in $D = 6$ or type IIB supergravity. If the multiplet has an off-shell formulation with some auxiliary fields, this off-shell multiplet is found as the cohomolgy of Q [25, 26] . In such cases, the pure superspinor complex only contains ghosts and fields, and antifields to these are found in a conjugate pure spinor superfield [26]. In many cases, one needs to use pure spinor superfields in nontrivial modules [27-32] (see, for example, the field Φ^a in section "Geometry vs. 4-Form"). In the language of Ref. [16], they belong to sections of some sheaf over the pure spinor space.

任何超多重态都可以推导为纯自旋超场的上同调。在许多情况下，零模上同调满足: 对应场定义了分量 BV 复形，并且可以定义积分。不满足这种情况的例子包括超多重态中包含自对偶张量的情形，例如 $N = (2,0)$ 多重态 (出现在 $D = 6$ 中) 或 IIB 型超引力。如果多重态存在带辅助场的脱壳构造，那么这个脱壳多重态就是 Q [25, 26] 的上同调。在这类情况下，纯自旋子复形仅包含鬼场和场，而它们的反场存在于共轭纯自旋超场中 [26]。许多情况下，我们需要使用非平凡模中的纯自旋超场 [27-32](例如参见“几何 vs. 四元形式”一节中的场 Φ^a)。用参考文献 [16] 的语言来说，它们属于纯自旋空间上某层的截面。

Higher derivative deformations of supersymmetric models, for example, super-symmetric Born-Infeld theory, may be given simple (polynomial) actions in pure spinor superfield theory [27,33].

超对称模型的高导数形变，例如超对称博恩-因费尔德理论，都可以在纯自旋 or 超场理论中给出简单的多项式作用量 [27,33]。

Supergravity, in particular in 11 dimensions, will be addressed in the following section:

超引力，尤其是 11 维超引力，将在下一节讨论:

Pure Spinor Partition Functions and Superalgebras

纯旋子配分函数与超代数

Given the usefulness of pure spinors for the description of supermultiplets in general, it seems meaningful to pursue a deeper mathematical investigation of the algebraic properties of pure spinor space itself. Functions on pure spinor space can be thought of algebraically as power series in λ , modulo the ideal generated by $(\lambda\gamma^a\lambda)$. The partition function of Eqs. (9) and (12) can indeed be understood as the partition function of the on-shell super-Yang-Mills multiplet by factoring out also a level 2 vector:

鉴于纯旋子对描述一般超多重态十分有用，对纯旋子空间本身的代数性质开展更深入的数学研究是很有意义的。纯旋子空间上的函数可以从代数上视作 λ 中的幂级数，模去 $(\lambda\gamma^a\lambda)$ 生成的理想。确实，通过分解出一个 2 级向量，式 (9) 和式 (12) 的配分函数可以被理解为壳外超杨-米尔斯多重态的配分函数:

$$Z(t) = (1-t)^{-(00001)} \otimes \left(1-t^2\right)^{(10000)} \tag{18}$$

$$\otimes \left[(00000) \oplus \bigoplus_{n=0}^{\infty} \left((n0010)\, t^{3+2n} \ominus (n1000)\, t^{4+2n}\right)\right].$$

The factor in the square represents the ghost zero-mode and the on-shell n’ th derivative of the fermion and the field strength. This is for the $D = 10$ super-Yang-Mills example. Similar statements hold for any multiplet. The first two factors are cancelled by the partition functions for functions of θ and x . In this way, it becomes clear that the pure spinor entirely encodes a full supermultiplet.

方框中的因子代表鬼零模，以及费米子和场强的壳上 n 阶导数。这是针对 $D=10$ 超杨-米尔斯的例子。任意多重态都有类似结论。前两个因子被 θ 和 x 函数的配分函数抵消。由此可见纯旋子完全编码了整个超多重态。

The investigation of the partition function of a pure spinor through the ghost structure associated with the bilinear constraint was initiated by Chesterman [34] and refined by Berkovits and Nekrasov [35]. Consider the BRST operator q for the pure spinor constraint. It will (generically) involve an infinite number of ghosts due to the infinite reducibility of the constraint. One may think of q as the coalgebra differential of a superalgebra, and the content of the algebra as a vector space may be deduced from a continued factorization of the partition function

通过双线性约束关联的鬼结构研究纯旋子配分函数的工作由 Chesterman[34] 开创，后由 Berkovits 与 Nekrasov[35] 完善。考虑纯旋子约束的 BRST 算符 q 。由于该约束具有无穷可约性，它 (一般地) 会包含无穷多个鬼。可以将 q 视作超代数的余代数微分，代数作为向量空间的内容可以通过对配分函数做连续分解推导得到

$$Z(t)=\prod_{n=1}^{\infty}(1-t^n)^{R_n} \tag{19}$$

One has to remember that statistics are switched and modules are conjugated when going from the ghosts (coalgebra elements) to the superalgebra. The superalgebra in question, which is our definition of the Koszul dual to the functions of a pure spinor, will always be some deformation of the direct sum of the supersymmetry algebra (levels 1 and 2) and the freely generated algebra on the supermultiplet (levels $n\geq 3$) [17]. The Koszul duality can be interpreted as a denominator formula for the superalgebra. In cases where the superfield is not a scalar, this is expected to generalize to character formulas for representations of the superalgebra.

必须记住，从鬼 (余代数元素) 过渡到超代数时，统计性质会发生切换，模也会取共轭。我们所讨论的这个超代数，即我们定义的纯旋子函数的 Koszul 对偶，它始终是超对称代数 (1 级和 2 级) 与超多重态上自由生成的代数 ($n\geq 3$ 级) 的直和的某种形变 [17]。Koszul 对偶可以解释为超代数的分母公式。当超场不是标量时，这可以推广为超代数表示的特征标公式。

When the constraint puts λ in a minimal orbit, the superalgebra is a Lie superalgebra, more precisely a Borcherds superalgebra [36]. For the particular case of $D=10$ super-Yang-Mills theory, the corresponding Borcherds superalgebra in fact exactly encodes the structure of interacting super-Yang-Mills theory [23]. This is a quite amazing and unexpected result, since all that is described by the cohomology is the linear multiplet. It is not yet clear what the corresponding statement is for other theories, e.g., $D=11$ supergravity, but partial results exist [17,37]. There, the superalgebra is not a Lie superalgebra, but and L_∞ algebra involving (at least) a three-bracket and a four-bracket.

当约束将 λ 置于极小轨道时，该超代数是一个李超代数，更准确地说是博彻兹超代数 [36]。对于 $D=10$ 超杨-米尔斯理论的特殊情形，对应的博彻兹超代数实际上完全编码了相互作用超杨-米尔斯理论的结构 [23]。这是一个非常惊人且出乎意料的结果，因为上同调描述的全部内容只是线性多重态。目前还不清楚其他理论 (例如 $D=11$ 超引力) 对应的结论是什么，但已有部分结果 [17,37]。在这些理论中，该超代数不是李超代数，而是 (至少) 包含一个三括号和一个四括号的 L_∞ 代数。

$D = 11$ Supergravity

$D = 11$ 超引力

We will not turn to $D = 11$ supergravity [38]. The pure spinor superfield formulation of this model can be derived from its traditional superspace [39, 40] formulation [41,42] in much the same way as the super-Yang-Mills theory was in section ”Pure Spinor Superfield Theory,” however with some additional ingredients.

我们接下来讨论 $D = 11$ 超引力 [38]。该模型的纯旋子超场表述可以从它的传统超空间 [39, 40] 表述 [41,42] 推导出来，推导方式和“纯旋子超场理论”一节中推导超杨-米尔斯理论的方式大致相同，只是需要补充一些额外内容。

Recall the component field content of the $D = 11$ supergravity multiplet: the metric g_{mn} , a three-form C with a four-form field strength $H = dC$, and the gravitino field $\chi_m{}^\alpha$ with field strength $\psi_{mn}{}^\alpha$. An essential feature that was used as a guideline for the construction of the supersymmetric action is that supersymmetry demands the presence of a Chern-Simons term $\int C \wedge H \wedge H$.

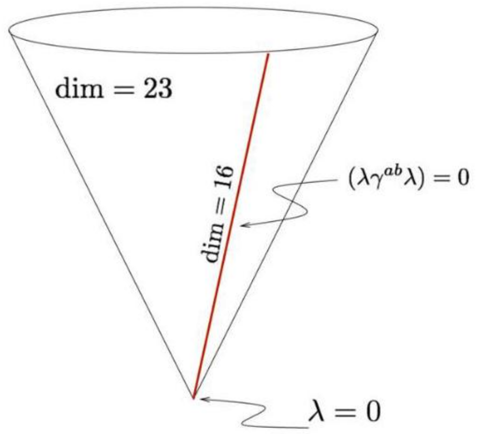
回顾一下 $D = 11$ 超引力多重态的分量场内容: 度规 g_{mn} 、一个带有四形式场强 $H = dC$ 的三形式 C ，以及 gravitino 场 (引力微子) $\chi_m{}^\alpha$ 和它的场强 $\psi_{mn}{}^\alpha$ 。超对称要求存在一个陈-西蒙斯项 $\int C \wedge H \wedge H$ ，这个关键性质是构造超对称作用量的指导原则。

In what follows, we will use 11-dimensional ”pure spinors” λ^α . A Dirac spinor in $D = 11$ has 32 components. The symmetric spinor bilinears consist of a vector, a 2-form and a 5-form, constructed with $\gamma^a_{\alpha\beta}, \gamma^{ab}_{\alpha\beta}$, and $\gamma^{abcde}_{\alpha\beta}$. A spinor subject to $(\lambda\gamma^a\lambda) = 0$ is not necessarily in a minimal orbit, which would require also $(\lambda\gamma^{ab}\lambda) = 0$. Rather, the pure spinor space consists of a ”generic,” 23-dimensional part, complemented by the 16-dimensional minimal orbit, which is a singular subspace, and the zero orbit, the tip of the cône. The space is sketched in Fig. 1.

下文我们将使用 11 维“纯旋子”λ^α 。$D = 11$ 中的狄拉克旋子有 32 个分量。对称旋子双线性由通过 $\gamma^a_{\alpha\beta}, \gamma^{ab}_{\alpha\beta}$ 和 $\gamma^{abcde}_{\alpha\beta}$ 构造的一个矢量、一个二形式和一个五形式组成。满足 $(\lambda\gamma^a\lambda) = 0$ 条件的旋子不一定处于极小轨道，极小轨道还需要满足条件 $(\lambda\gamma^{ab}\lambda) = 0$ 。实际上，纯旋子空间由“一般”的 23 维部分构成，辅以作为奇异子空间的 16 维极小轨道，以及处于锥顶的零轨道。该空间的示意图见图 1。

Fig. 1 A sketch of the space of $D = 11$ pure spinors

图 1 $D = 11$ 纯旋子空间示意图



Geometry vs. 4-Form

几何方法与 4-形式方法对比

There are two different versions of superfields that can describe the on-shell (linearized) supergravity multiplet. One relies on the standard description of superspace geometry, where one introduces a dynamical super-vielbein $E_M{}^A$. Then, conventional constraints are used to eliminate all components except the lowest-dimensional ones, $E_\mu{}^a$ in a controlled and covariant way [43-45]. Note that physical fields then are described by one-forms (in fermionic indices), but with an extra index a , while superdiffeomorphisms can be thought of as sitting in a superfield ξ^a with the bosonic diffeomorphism parameters as leading components. The situation reminds of the treatment of super-Yang-Mills theory in the previous section, although all fields have an extra index a . It can indeed be verified that the linearized (around Minkowski space) multiplet is described by the cohomology of $Q = (\lambda D)$ on a field $\Phi^a(x, \theta, \lambda)$. The field is in addition required to have a "shift symmetry" [27, 46, 47] $\Phi^a \simeq \Phi^a + (\lambda\gamma^a\varrho)$ for an arbitrary parameter $\varrho^\alpha(x, \theta, \lambda)$. (The shift symmetry ties together the index structure with the cohomology and is also directly responsible for the presence of the fermionic diffeomorphism ghosts in the zero-mode cohomology.)

存在两种不同的超场版本可描述在壳 (线性化) 超引力多重态。一种依赖超空间几何的标准描述: 引入动力学超标架 $E_M{}^A$ ，随后以协变可控的方式利用常规约束消去除最低维分量外的所有分量 $E_\mu{}^a$ 。需注意, 物理场由 (费米指标下的)1-形式描述, 但带有额外指标 a , 而超微分同胚可理解为包含在以玻色微分同胚参数为首分量的超场 ξ^a 中。这种情况类似上一节对超杨-米尔斯理论的处理，区别在于所有场都多了一个额外指标 a。我们确实可以验证, 围绕闵氏空间线性化的多重态由场 $\Phi^a(x, \theta, \lambda)$ 上 $Q = (\lambda D)$ 的上同调描述。此外该场还需要满足“平移对称性”[27, 46, 47] $\Phi^a \simeq \Phi^a + (\lambda\gamma^a\varrho)$ ，其中 $\varrho^\alpha(x, \theta, \lambda)$ 为任意参数。(平移对称性将指标结构与上同调联系起来，也是零模上同调中存在费米微分同胚鬼的直接原因。)

The above is one way to relate the on-shell linearized supergravity multiplet to pure spinor superfield cohomology. Since it is geometrical, it carries no information about the gauge symmetry of the 3-form C , which indeed only appears through its field strength $H\widehat{A}$ in the dimension 1 torsion. The other way of reproducing the linearized multiplet is through a scalar field. The full ghost system for the C field contains a ghost, a ghost for ghost, and a ghost for ghost for ghost. The latter is a fermionic 0 -form. We can think of it as the θ - and λ -independent zero-mode cohomology of a pure spinor superfield Ψ of ghost number 3 and dimension -3. A careful calculation of the zero-mode cohomology gives at hand that it indeed contains the mentioned ghosts, together with the superdiffeomorphism ghosts (at λ^2), the physical fields (at λ^3), and all corresponding antifields. We refer to Refs. [1, 46, 47] for the detailed calculations.

以上是将在壳线性化超引力多重态与纯旋量超场上同调联系起来的一种方式。由于它是几何方法，不包含 3-形式 C 规范对称性的任何信息——3-形式确实仅通过其场强 $H\widehat{\mathrm{A}}$ 出现在 1 维挠率中。重构线性化多重态的另一种方式是通过标量场。C 场的完整鬼系统包含鬼、鬼的鬼、鬼的鬼的鬼，其中最后一个是费米 0-形式。我们可以将其理解为鬼数为 3、维数为-3 的纯旋量超场 Ψ，其零模上同调不依赖 θ 和 λ。仔细计算零模上同调后可知，它确实包含上述鬼，加上 (λ^2 处的) 超微分同胚鬼、(λ^3 处的) 物理场以及所有对应的反场。详细计算可参考文献 [1, 46, 47] 。

Now, we are in a situation where the traditional supergeometric approach gives the full nonlinear equations of motion, but does not account for the full ghost structure. The scalar field Ψ , on the other hand, is more fundamental in that it contains all ghosts, and also the potential C , but it is a priori unclear how to go beyond the linearized level. Importantly, in order to write down an action containing the Chern-Simons term, Ψ is needed.

目前的情况是: 传统超几何方法能给出完整的非线性运动方程，但无法给出完整的鬼结构；另一方面，标量场 Ψ 更基本，它包含所有鬼，也包含势场 C，但先验上并不清楚如何推广到线性化以上的层次。关键在于，要写出包含陈-西蒙斯项的作用量，必须用到 Ψ 。

BV Action

BV 作用量

Before giving the form of the full nonlinear action, we need to understand integration, regularization, etc. in a way analogous to the 10-dimensional case. We will refrain from detailed expressions. The pure spinor space is 23-dimensional. We again introduce non-minimal variables $\bar{\lambda}$ and $d\bar{\lambda}$ and include the Dolbeault operator $\bar{\partial}$ in Q . The top cohomology of the third-order ghost antifield now sits at $\lambda^7\theta^9$. A measure based on this cohomology has the correct ghost number -7 for an action with $\int \Psi Q \Psi$, where Ψ carries ghost number 3 . The pure spinor space is again Calabi-Yau, with $\Omega \sim \lambda^{-7}(d\lambda)^{23}$. A completely analogous regularization will contain $\theta^{23}\left(d\bar{\lambda}\right)^{23}$, so the θ integration will effectively pick out a term with θ^9 $(9 = 32 - 23)$, as desired. A linearized action

在给出完整非线性作用量的形式之前，我们需要类比 10 维情形理解积分、正则化等内容。我们不会给出详细表达式。纯旋量空间是 23 维的。我们再次引入非最小变量 $\bar{\lambda}$ 和 $d\bar{\lambda}$，并将 Dolbeault 算符 $\bar{\partial}$ 包含在 Q 中。三阶鬼反场的上同调现在位于 $\lambda^7\theta^9$ 。基于该上同构造的测度具有正确的鬼数-7，适用于含 $\int \Psi Q \Psi$ 的作用量，其中 Ψ 的鬼数为 3。纯旋量空间仍是 Calabi-Yau 空间，满足 $\Omega \sim \lambda^{-7}(d\lambda)^{23}$ 。采用完全类似的正则化会得到 $\theta^{23}\left(d\bar{\lambda}\right)^{23}$，因此 θ 积分会如预期般有效选出含 θ^9 $(9 = 32 - 23)$ 的项。线性化作用量

$$S_2 = \frac{1}{2}\int [dZ]\, \Psi Q \Psi \tag{20}$$

reproduces the on-shell multiplet correctly.

可以正确给出在壳多重态。

How are interactions constructed as additional terms in a BV action? One starting point may be to look at the Chern-Simons term $\int C \wedge H \wedge H$. It must contain at least one field Ψ , but the remaining factors can in principle be formed from Φ^a , containing the field strength H . The concrete task now becomes to find an expression for Φ^a in terms of Ψ , such that cohomology maps to cohomology. This means that one needs to find a bosonic operator R^a of ghost number -2 and dimension 2 which commutes with Q modulo terms of the type $(\lambda\gamma^a\varrho)$. The procedure is similar to that of finding a b operator, used in gauge fixing (see section "Quantum Theory"). Such an operator was constructed in Ref. [46] using non-minimal variables. It takes a somewhat complicated form, beginning as

相互作用如何作为 BV 作用量的附加项构造？一个起点可以是考察陈-西蒙斯项 $\int C \wedge H \wedge H$ 。它必须至少包含一个场 Ψ ，但剩余因子原则上可以由包含场强 H 的 Φ^a 构造。当前的具体任务是找到 Φ^a 用 Ψ 表示的表达式，使得上同调映射到上同调。这意味着需要找到一个鬼数为-2、量纲为 2 的玻色算符 R^a ，它与 Q 对易，余项形如 $(\lambda\gamma^a\varrho)$ 。该过程类似寻找规范固定中用到的 b 算符 (参见"量子理论"一节)。文献 [46] 已经利用非最小变量构造出了这样一个算符，其形式较为复杂，开头为

$$R^a = \left((\lambda\gamma_{cd}\lambda)\left(\bar\lambda\gamma^{cd}\bar\lambda\right)\right)^{-1}\left(\bar\lambda\gamma^{ab}\bar\lambda\right)\partial_b + \ldots \tag{21}$$

One can then use Ψ as the fundamental field and write $\Phi^a = R^a\Psi$. A term

接下来可以将 Ψ 作为基本场写出 $\Phi^a = R^a\Psi$ 。这样一个项

$$S_3 = \frac{1}{6}\int [dZ]\,(\lambda\gamma_{ab}\lambda)\,\Psi R^a\Psi R^b\Psi \tag{22}$$

is then guaranteed to fulfill $(S_2, S_3) = 0$, i.e., work as a linear deformation of S_2 [46]. Note that the factor $(\lambda\gamma_{ab}\lambda)$ serves several purposes: It contracts the indices on a pair of fermionic fields. It ensures the correct ghost number and dimensions of the interaction term. And finally, it ensures the invariance under the shift symmetry, thanks to the Fierz identity $\left(\gamma^b\lambda\right)_\alpha(\lambda\gamma_{ab}\lambda) = 0$, which holds for pure spinors. It can be verified, using explicit expressions for the cohomologies, that the Chern-Simons term is correctly reproduced by S_3 .

就必然满足 $(S_2, S_3) = 0$ ，即可以作为 S_2 的线性形变生效 [46]。注意因子 $(\lambda\gamma_{ab}\lambda)$ 有多重作用: 它收缩一对费米场的指标，保证相互作用项具有正确的鬼数和量纲，最后还得益于纯旋子满足的费尔兹恒等式 $\left(\gamma^b\lambda\right)_\alpha(\lambda\gamma_{ab}\lambda) = 0$ ，保证了平移对称性下的不变性。利用上同调的显式表达式可以验证，陈-西蒙斯项确实可以由 S_3 正确得到。

In order to construct a complete action, the master equation $(S, S) = 0$ must be checked, not only to linear order in S_3 as above. It turns out [47] that only a minor modification is needed: a four-point coupling which is almost of the same form as S_3 . It relies on yet another operator, T , of ghost number -3 and dimension 3 . The field $T\Psi$ then carries ghost number and dimension 0, and its ghost number 0 part can be thought of as containing the trace of the linearized gravity field. Then, the action

为了构造完整作用量，必须验证主方程 $(S,S)=0$ ，而不仅仅是上述 S_3 的线性阶。结果表明 [47]，只需要做微小修改: 一个四点耦合，其形式几乎和 S_3 完全一致。它依赖于另一个鬼数-3、量纲 3 的算符 T 。场 $T\Psi$ 的鬼数和量纲均为 0，其鬼数 0 部分可以理解为包含线性化引力场的迹。那么作用量

$$S=\int[dZ]\left(\frac{1}{2}\Psi Q\Psi+\frac{1}{6}(\lambda\gamma_{ab}\lambda)\left(1-\frac{3}{2}T\Psi\right)\Psi R^a\Psi R^b\Psi\right) \tag{23}$$

turns out to satisfy $(S,S)=0$ to all orders. It is striking, but ideal from the point of view of perturbative calculations, that a model containing gravity becomes polynomial around Minkowski space. A detailed understanding, e.g., through homotopy transfer, of how the non-polynomial nature of geometry around Minkowski space arises, is still lacking. Neither does the construction offer any direct clues concerning how to proceed to other backgrounds. Some remarks concerning these issues are given in the concluding section.

结果表明它对所有阶都满足 $(S,S)=0$ 。一个包含引力的模型在闵可夫斯基空间周围是多项式的，这十分引人注意，但从微扰计算的角度来看非常理想。目前仍缺乏对闵可夫斯基空间周围几何的非多项式性质如何产生的详细理解，例如通过同伦传递的方式。该构造也没有为如何推广到其他背景提供任何直接线索。结语部分对这些问题给出了一些讨论。

Twisting

扭转变换

Pure spinor superfields provide a good framework for twisting supersymmetric theories and to find all possible twistings [48,49]. This is because any point in the space of spinors obeying $(\lambda\gamma^a\lambda)=0$ provides a nilpotent operator $\lambda^\alpha D_\alpha$. (Note that here λ is not a variable but takes some specific value.) The list of possible twistings can be read off from the stratification of pure spinor space in different orbits under the Lorentz group, forming subspaces of pure spinor space.

纯旋量超场为扭转变换超对称理论、找出所有可能的扭转变换提供了良好框架 [48,49]。这是因为满足 $(\lambda\gamma^a\lambda)=0$ 的旋量空间中的任意点都给出一个幂零算子 $\lambda^\alpha D_\alpha$ 。(注意此处 λ 不是变量，而是取某个特定值。) 所有可能的扭转变换可从纯旋量空间在洛伦兹群下不同轨道的分层得到，这些轨道构成纯旋量空间的子空间。

In supergravity, supersymmetry is local, and twisting is performed by giving an expectation value to a superdiffeomorphism ghost [50]. A treatment in the pure spinor formalism is favorable, since these ghosts are naturally present. Among other theories, the twistings of $D=11$ supergravity have been thus examined [49, 51, 52] . The minimal twist leads to the $SL(5)$ supersymmetric model of Ref. [32].

在超引力中，超对称是定域的，扭转变换通过给超微分同胚鬼赋予期望值来实现 [50]。在纯旋量形式体系中处理该问题更有利，因为这些鬼自然存在于该体系中。在其他理论之外，人们还借此研究了 $D=11$ 超引力的扭转变换 [49, 51, 52] 。极小扭转变换导出了文献 [32] 中的 $SL(5)$ 超对称模型。

Superstrings

超弦

The covariant quantization of space-time supersymmetric string theory remained elusive for a long time, until Berkovits constructed the pure spinor superstring [53-56]. The variables used are the same as displayed above for $D = 10$ super-Yang-Mills theory. In both the left- and right-moving sectors of the world sheet, one introduces in addition to the superspace coordinates X (self-conjugate) and θ , with its conjugate p , a pure spinor λ and its conjugate ω . The variable λ has the same chirality as θ . In type IIA superstring theory, this chirality is opposite for left and right movers and in type IIB the same.

时空超对称弦理论的协变量化长期以来都未能实现，直到 Berkovits 构造了纯旋量超弦 [53-56]。所用变量与上文 $D = 10$ 超杨-米尔斯理论给出的一致。在世界面的左行和右行区，除了超空间坐标 X (自共轭) 和 θ 及其共轭 p 之外，还引入了纯旋量 λ 及其共轭 ω 。变量 λ 与 θ 手征相同。在 IIA 型超弦理论中，左行和右行的手征相反，而在 IIB 型中手征相同。

The left-moving BRST operator reads

左行 BRST 算符为

$$Q = \oint \lambda^\alpha d_\alpha \tag{24}$$

where

其中

$$d_\alpha = p_\alpha + \partial X^a (\gamma_a \theta)_\alpha + \frac{1}{8} (\gamma^a \theta)_\alpha (\theta \gamma_a \partial \theta) , \tag{25}$$

with the operator product expansion

满足算符乘积展开

$$d_\alpha (z) \, d_\beta (\zeta) = \frac{1}{z - \zeta} \gamma_{a \alpha \beta} \Pi^a + \text{ (regular)}, \tag{26}$$

where $\Pi^a = \partial X^a - (\theta \gamma^a \partial \theta)$ is the momentum conjugate to X in the Green-Schwarz superstring. This implies $Q^2 = 0$. Again, it can of course be extended with nonminimal variables.

式中 $\Pi^a = \partial X^a - (\theta \gamma^a \partial \theta)$ 是 Green-Schwarz 超弦中对应 X 的共轭动量，由此可得 $Q^2 = 0$ 。当然，该形式也可以扩展加入非最小变量。

Notably the list of fields above is complete, including ghosts. There is no Virasoro ghost pair (b, c) (and, unlike the Neveu-Schwarz-Ramond superstring, no super-Virasoro ghosts (β, γ)). This of course also happens for the superparticle. All ”coordinates” are world-sheet scalars. The cancellation of the conformal anomaly requires no Virasoro ghost but simply reads $c = 10 - 2 \cdot 16 + 2 \cdot 11 = 0$. This may seem as a simplification but also has its price in making, e.g., gauge fixing more complicated (see section ”Quantum Theory”).

值得注意的是，上文包含鬼场在内的场列表已经是完整的。这里不存在 Virasoro 鬼对 (b, c) ，且与 Neveu-Schwarz-Ramond 超弦不同，也不存在超 Virasoro 鬼 (β, γ) 。超粒子当然也是如此。所有“坐标”都是世界面标量。共形反常抵消不需要 Virasoro 鬼，条件仅为 $c = 10 - 2 \cdot 16 + 2 \cdot 11 = 0$ 。这看似简化，但也有代价，例如会让规范固定变得更复杂 (参见“量子理论”一节)。

Integration over pure spinor variables follows the same principles as for the super-Yang-Mills theory.

对纯旋量变量的积分遵循与超杨-米尔斯理论相同的原理。

Quantum Theory

量子理论

The procedures sketched in this section focus on principles and qualitative results. The issue of gauge fixing and the b operator is discussed in somewhat more details, since this is one of the points where the formalism becomes complicated and simplifications are desired. The physical fields are ”hidden” within a structure which exhibits many qualitatively simple features. Their extraction from that structure is more complicated [16, 24]. If one wants to use the formalism to derive precise quantitative results, much work is involved (see references below).

本节概述的流程着重于原理与定性结果。规范固定问题和 b 算子会得到稍详细的讨论，因为这是形式体系变得复杂、需要引入简化的关键点之一。物理场“隐藏”在一个具备许多定性简单特征的结构中。从该结构中提取物理场的过程更为复杂 [16, 24]。若要使用该形式体系推导精确的定量结果，仍需完成大量工作 (见下文参考文献)。

Gauge Fixing

规范固定

The non-minimal variables open for a possibility to construct operators with negative ghost number. The so-called b ghost or b operator is the standard example (see also the negative ghost number operators of section ” $D = 11$ Supergravity”). It is called b because it assumes the rôle of the conjugate to the ghost c for world-line reparametrizations or world-sheet conformal transformations (see the cancellation of the conformal anomaly in section ”Superstrings”). In pure spinor superfield theory, it is a composite operator. This is because ” $p^2 = 0$ ” (in a superparticle action) is only a derived linearized equation of motion, a consequence (after gauge fixing) of $Q\Phi = 0$, not a constraint associated with a world-line symmetry.

非最小变量为构造负鬼数算子提供了可能。所谓的 b 鬼或 b 算子就是标准例子 (另见“$D = 11$ 超引力”章节的负鬼数算子)。它被称为 b ，是因为它充当世界线重参数化或世界片共形变换中鬼场 c 的共轭量 (参见“超弦”章节中共形反常的抵消)。在纯旋子超场理论中，它是一个复合算子。这是因为“$p^2 = 0$ ”(在超粒子作用量中) 仅仅是导出的线性化运动方程，是规范固定后 $Q\Phi = 0$ 的推论，而非与世界线对称性关联的约束条件。

In order to perform perturbative quantum calculations, gauge fixing is necessary. The ”kinetic operator” Q is of course not invertible. If one can find an operator b such that $\{Q, b\} = \square$ and chooses the Siegel gauge [57]

为了进行微扰量子计算，规范固定是必要的。“动能算子” Q 当然是不可逆的。如果能找到一个算子 b 满足 $\{Q, b\} = \square$ ，并选取西格尔规范 [57]

$$b\Psi = 0, \tag{27}$$

the propagator G can be written as

则传播子 G 可以写为

$$G = \frac{b}{\square}. \tag{28}$$

In the following, we will write the field theory b operator. The one for string theory is very similar (in the same way as the Q s of Eqs. (2) and (24) are), just containing a small number of more terms with derivatives. As mentioned, this is one of the instances where things become complicated in the pure spinor formalism.

下文我们将写出场论的 b 算子。弦论的对应形式非常相似 (和式 (2) 与式 (24) 中的 Q 情况一样)，仅多了少数几个含导数的项。如前所述，这是纯旋子形式论中问题变复杂的情况之一。

The b operator in $D = 10$ was constructed in Ref. [22] using non-minimal variables and reads

$D = 10$ 中的 b 算子由文献 [22] 利用非最小变量构造，形式为

$$\begin{aligned} b &= b_0 + b_1 + b_2 + b_3 \\ &= -\frac{1}{2}\left(\lambda\bar{\lambda}\right)^{-1}\left(\bar{\lambda}\gamma^a D\right)\partial_a \\ &+\frac{1}{16}\left(\lambda\bar{\lambda}\right)^{-2}\left(\bar{\lambda}\gamma^{abc}d\bar{\lambda}\right)\left[N_{ab}\partial_c - \frac{1}{24}\left(D\gamma_{abc}D\right)\right] \\ &+\frac{1}{64}\left(\lambda\bar{\lambda}\right)^{-3}\left(d\bar{\lambda}\gamma^{abc}d\bar{\lambda}\right)\left(\bar{\lambda}\gamma_a D\right)N_{bc} \\ &-\frac{1}{1024}\left(\lambda\bar{\lambda}\right)^{-4}\left(\bar{\lambda}\gamma^{ab}{}_i d\bar{\lambda}\right)\left(d\bar{\lambda}\mathfrak{g}^{cdi}d\bar{\lambda}\right)N_{ab}N_{cd}, \end{aligned} \tag{29}$$

where $N = (\lambda\omega)$ and $N_{ab} = (\lambda\gamma_{ab}\omega)$ are invariant operators, in the sense that they respect the pure spinor constraint $(\lambda\gamma^a\lambda) = 0$.

其中 $N = (\lambda\omega)$ 和 $N_{ab} = (\lambda\gamma_{ab}\omega)$ 都是不变算子，即它们满足纯旋子约束 $(\lambda\gamma^a\lambda) = 0$ 。

The b operator in $D = 11$ [58, 59] is somewhat more complicated. We will not display the full expression, but note that it is singular on the 16-dimensional subspace (like the negative ghost number operators encountered in section ” $D = 11$ Supergravity”) and begins as

$D=11$ [58, 59] 中的 b 算子要更复杂一些。我们不给出完整表达式，仅需指出它在 16 维子空间上是奇异的 (就像“$D=11$ 超引力”章节中遇到的负鬼数算子一样)，其形式开头为

$$b=\left((\lambda\gamma_{de}\lambda)\left(\bar{\lambda}\gamma^{de}\bar{\lambda}\right)\right)^{-1}\left(\bar{\lambda}\gamma_{ab}\bar{\lambda}\right)\left(\lambda\gamma^{ab}\gamma^{c}D\right)\partial_{c}+\ldots \tag{30}$$

There is also a possibility to find a b operator that acts within functions of the minimal variables [60], using the principles of Ref. [61]. On (holomorphic) functions of a $D=10$ pure spinor λ , the ”invariant derivative operator”

也可以利用文献 [61] 的原理，得到作用在最小变量函数上的 b 算子 [60]。对于 $D=10$ 纯旋子 λ 的 (全纯) 函数，“不变导数算子”

$$\tilde{\omega}_{\alpha}=\omega_{\alpha}-\frac{1}{4(N+3)}(\gamma^{a}\lambda)_{\alpha}(\omega\gamma_{a}\omega) \tag{31}$$

acts exactly like ω_α between monomials $\lambda^{\alpha_1}\ldots\lambda^{\alpha_p}$ and annihilates the ideal generated by $(\lambda\gamma^a\lambda)$. The minimal b' operator reads

在单项式 $\lambda^{\alpha_1}\ldots\lambda^{\alpha_p}$ 之间的作用完全等价于 ω_α ，并且零化 $(\lambda\gamma^a\lambda)$ 生成的理想。最小形式的 b' 算子为

$$b'=\frac{1}{(N+4)(N+5)(N+6)}\left[\frac{1}{2}\left(N^{2}+9N+15\right)(\tilde{\omega}\gamma^{a}D)\,\partial_{a}-\frac{1}{128}N^{ab}\left(\tilde{\omega}D^{3}_{ab}\right)\right],$$

(32)

where D^3_{ab} is the antisymmetric product of three D s in (01001),

其中 D^3_{ab} 是 (01001) 表示下三个 D 的反对称乘积，

$$\left(D^{3}\right)_{ab}^{\alpha}=\left(\gamma^{i}\right)^{\alpha[\beta}\left(\gamma_{abi}\right)^{\gamma\delta]}D_{\beta}D_{\gamma}D_{\delta}. \tag{33}$$

It can be shown explicitly that b and b' differ by a Q -exact expression. Using b' , it is seen directly that Siegel gauge implies Lorenz gauge for the Yang-Mills connection. Namely, acting on a ghost number 0 field $\Psi=\lambda^\alpha A_\alpha, b'\Psi=\frac{1}{16}\partial^a(D\gamma_a A)$. A similar minimal b operator should exist for any supersymmetric theory with local symmetries, e.g., $D=11$ supergravity, but has not been constructed.

可以直接证明 b 和 b' 仅相差一个 Q 恰当项。利用 b' 可以直接看出，西格尔规范对杨-米尔斯联络蕴含洛伦兹规范，即它作用在鬼数为 0 的场 $\Psi=\lambda^\alpha A_\alpha, b'\Psi=\frac{1}{16}\partial^a(D\gamma_a A)$ 上成立。对于任何存在局域对称性的超对称理论，例如 $D=11$ 超引力，都应当存在类似的最小 b 算子，但目前尚未构造出来。

It may rightly be claimed that gauge fixing, in the form presented here, is rudimentary, and more or less implemented at a first-quantized level. A proper field-theoretic BV gauge fixing [62], involving a gauge fixing fermion, has not been developed in pure spinor superfield theory.

可以合理地说，本文介绍的这种规范固定还比较初级，大体上是在第一量子化层面实现的。包含规范固定费米子的、恰当的场论 BV 规范固定，尚未在纯旋子超场理论中建立起来。

Perturbative Results

微扰结果

The construction sketched above gives a recipe for calculating scattering amplitudes in the pure spinor formalism. Any diagram - which will contain a large number of component field diagrams - should be saturated with appropriate vertex operators [63] representing external states. There is a remaining issue of regularization at $\lambda = 0$ which was addressed and solved in Ref. [64]. This is due to the b operators in propagators containing negative powers of $(\lambda\bar{\lambda})$ that ultimately risk to make integrals divergent. Explicit evaluation of the regulated integrals is in general extremely complicated. Results exist for superstring theory on Minkowski space [63-67] and on anti-de Sitter space [68,69].

上文概述的构造给出了纯旋子形式下计算散射振幅的一套方法。任何图 (包含大量分量场图) 都应当用代表外部态的合适顶点算符 [63] 饱和化。在 $\lambda = 0$ 处存在规范正则化的遗留问题，该问题已在文献 [64] 中得到讨论和解决。问题源于传播子中的 b 算符含有 $(\lambda\bar{\lambda})$ 的负幂次，最终可能导致积分发散。一般来说，对正则化后积分的显式计算极其复杂。目前已有闵氏空间上 [63-67] 和反德西特空间上 [68,69] 超弦理论的相关结果。

The degree of convergence of loop diagrams in pure spinor superfield theory is generically much better than for loop diagrams in a component formulation or with superfields manifesting some fraction of supersymmetry. Typical behavior is the vanishing of bubbles and triangles in off-shell diagrams, i.e., as subdiagrams of any diagram. For maximal super-Yang-Mills theory in $D = 4\,[70, 71]$, power counting is enough to demonstrate perturbative finiteness. In maximal supergravity in $D = 4$ [58,72-74], power counting shows finiteness up to 6 loops and possibly a divergence at 7 loops (see also Refs. [75-77]). The precise statement is that an L -loop diagram is convergent in D dimensions if $D < 2 + \frac{14}{L}$, while for super-Yang-Mills theory, it reads $D < 4 + \frac{6}{L}$.

纯旋子超场理论中圈图的收敛程度，通常远好于分量表述或仅显现部分超对称的超场表述下的圈图。典型表现是离壳图 (即任意图的子图) 中的泡图和三角图都为零。对于 $D = 4\,[70, 71]$ 维的最大超对称杨-米尔斯理论，幂计数就足以证明其微扰有限性。对于 $D = 4$ 维的最大超引力 [58,72-74]，幂计数显示该理论在至多 6 圈时是有限的，在 7 圈处可能发散 (另见文献 [75-77])。精确表述为: 当满足 $D < 2 + \frac{14}{L}$ 时，D 维中的 L 圈图收敛，而对于超对称杨-米尔斯理论，收敛条件为 $D < 4 + \frac{6}{L}$ 。

Remarks

评论

Some final remarks concern shortcomings of the present status of pure spinor superfield theory and some desirable developments.

最后的几点评论讨论纯旋子超场理论当前存在的不足，以及若干值得开展的研究方向。

The classical theory of pure spinor superfields exhibits a striking simplicity. Quantum calculations tend to become cumbersome, although in principle well defined, mainly due to the complicated expression for the b operator used in gauge fixing and the regularization it brings along. It remains an open question if these calculations can be simplified, either by finding a replacement for the b operator or by some completely different means.

纯旋子超场的经典理论极具简洁性。量子计算虽然原则上定义良好，但往往十分繁琐，这主要是因为规范固定所用的 b 算符形式复杂，且该算符会引入复杂的正则化。无论是寻找 b 算符的替代方案，还是通过其他全新方法，这些计算能否得到简化目前仍是一个待解决的问题。

One approach, which has not been properly explored, would be to use the minimal (holomorphic) version of the b operator, b' of Eq. (32). The construction will certainly extend to other negative ghost number operators [60,61]. Possibly, in such a framework, the rôle of the non-minimal variables can be limited to integration, with the "simple" regularization of Eq. (15), and the complicated regularizations at $\lambda = 0$ may be avoided.

一种尚未得到充分研究的思路是采用 b 算符的极小 (全纯) 版本，即式 (32) 的 b' 。该构造显然可以推广到其他负鬼数算符 [60,61]。在该框架下，非极小变量的作用或许可以仅限于积分，采用式 (15) 的“简单”正则化，从而避免 $\lambda = 0$ 处的复杂正则化。

An urgent question for supergravity is the lack of manifest background invariance of the action (23). This is of course usual in string theory and string field theory [78] (see however Ref. [79]), but one should be able to do better in a supergravity theory. Indeed, even if the basis of the construction is in supergeometry, the geometric picture is lost in the final form. There is some hope for "re-geometrization" and for an understanding how to deform the model to non-flat backgrounds. It relies on deforming the algebra which is Koszul dual to functions of 11-dimensional pure spinors [17], in a manner similar to Ref. [80].

超引力领域的一个紧迫问题是，作用量 (23) 缺乏明显的背景不变性。这一点在弦论和弦场论中十分常见 [78](但参见文献 [79])，但在超引力理论中我们理应找到更优的方案。事实上，即便该构造的基础是超几何，最终形式也丢失了几何图像。我们仍有希望完成“重几何化”，并理解如何将该模型推广到非平坦背景。这依赖于对与 11 维纯旋子函数 Koszul 对偶的代数进行形变，思路与文献 [80] 类似。

As mentioned in section "Gauge Fixing," a proper field-theoretic BV gauge fixing procedure has not been developed for pure spinor superfield theory. There is no doubt that this can be done. It is probably one of the most important points on which the framework should be developed.

正如“规范固定”一节所述，纯旋子超场理论尚未建立 proper 的场论 BV 规范固定流程。毫无疑问这一步是可以完成的，这很可能是该框架最需要发展的方向之一。

The whole idea about the formalism presented is to manifest as much symmetry as possible. It is well known that dimensional reductions of $D = 11$ enjoy

本文介绍这套形式体系的核心思路是尽可能显化对称性。众所周知，$D = 11$ 的维度约化拥有

U-duality and that this symmetry can be "geometrized" within the context of exceptional geometry [81-84]. Can the pure spinor framework be extended to accommodate for these symmetries? Such a task may be very difficult, due to the infinite reducibility of local symmetries in extended geometry, since the pure spinor superfields always are based on the lowest-dimensional ghost field. Indeed, already double supergeometry [85,86] contains infinite reducibility in the Ramond-Ramond sector.

U 对偶，且该对称性可以在例外几何框架内实现“几何化”[81-84]。纯旋子框架能否拓展以容纳这些对称性？由于扩展几何中局部对称性存在无限可约性，而纯旋子超场始终基于最低维鬼场，因此这项工作可能十分困难。事实上，即便是双重超几何 [85,86]，其拉蒙德-拉蒙德 sector 也已经存在无限可约性。

Cross-References

交叉引用

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超弦的纯旋量表述及其应用

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简单超引力

- 11D Supergravity and Hidden Symmetries

- 11 维超引力与隐藏对称性

Supergravity Amplitudes, the Double Copy, and Ultraviolet Behavior

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